Exercise 19

Let $f(x) = \dfrac{x}{x-2}$. Find a function $y = g(x)$ so that $(f \circ g)(x) = x$.

Solution

To find this function, replace x with y, replace $f(x)$ with x, and solve for y.

$$x = \frac{y}{y-2}$$

$$x(y-2) = y$$

$$xy - 2x = y$$

$$xy - y = 2x$$

$$y(x-1) = 2x$$

$$y = \frac{2x}{x-1}$$

Therefore,

$$g(x) = \frac{2x}{x-1}.$$